3 EXACT ROOTS

Square root

The opposite of squaring a number is finding its square root.

$\sqrt{25} = 5$ because $5^2 = 25$

EXAMPLES

$\sqrt{400} = 20 \Leftrightarrow 20^2 = 400$

$\sqrt{\frac{4}{9}} = \frac{2}{3} \Leftrightarrow \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

1. Calculate.

a) $\sqrt{81}$ b) $\sqrt{900}$ c) $\sqrt{1\,600}$

d) $\sqrt{\frac{1}{4}}$ e) $\sqrt{\frac{4}{25}}$ f) $\sqrt{\frac{16}{49}}$

g) $\sqrt{0.01}$ h) $\sqrt{1.21}$ i) $\sqrt{0.0016}$

Cube root

The opposite of cubing a number is finding its cube root.

$\sqrt[3]{8} = 2$ because $2^3 = 8$

EXAMPLES

$\sqrt[3]{1{,}000} = 10 \Leftrightarrow 10^3 = 1{,}000$

$\sqrt[3]{\frac{1}{8}} = \frac{1}{2} \Leftrightarrow \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$\sqrt[3]{0.027} = 0.3 \Leftrightarrow 0.3^3 = 0.027$

2. Calculate.

a) $\sqrt[3]{64}$ b) $\sqrt[3]{125}$ c) $\sqrt[3]{27{,}000}$

d) $\sqrt[3]{\frac{1}{27}}$ e) $\sqrt[3]{\frac{8}{125}}$ f) $\sqrt[3]{\frac{125}{1{,}000}}$

g) $\sqrt[3]{0.001}$ h) $\sqrt[3]{0.064}$ i) $\sqrt[3]{0.216}$

Other roots

We can also use this same pattern to calculate roots with an index number greater than 3.

$\sqrt[4]{81} = 3$ because $3^4 = 81$

EXAMPLES

$\sqrt[4]{10{,}000} = 10 \Leftrightarrow 10^4 = 1{,}000$

$\sqrt[5]{32} = 2 \Leftrightarrow 2^5 = 32$

3. Calculate.

a) $\sqrt[4]{16}$ b) $\sqrt[5]{0.00032}$ c) $\sqrt[6]{1{,}000{,}000}$

d) $\sqrt[4]{\frac{1}{81}}$ e) $\sqrt[5]{\frac{32}{243}}$ f) $\sqrt[6]{\frac{1}{64}}$

4. Calculate.

a) $\sqrt[3]{7^3}$ b) $\sqrt[5]{10^5}$ c) $\sqrt[6]{8^6}$

d) $\sqrt[3]{5^6}$ e) $\sqrt[4]{3^8}$ f) $\sqrt[5]{7^{10}}$

4 RADICALS

What are radicals?

Expressions containing the root ("radical") symbol are called radicals.

INDEX → $\sqrt[3]{(x^2 \cdot y)^2}$ ← RADICAND

EXAMPLES

$3\sqrt{5}$ $\quad \dfrac{2\sqrt[3]{x}}{5}$ $\quad 6\sqrt[3]{3} - 5\sqrt{2}$

are radicals.

Adding radicals

Radicals can be added when they have the same index and the same radicand.

$3\sqrt{2} - \sqrt{2} + 5\sqrt{2} = 7\sqrt{2}$

$\sqrt{5} + \sqrt{3}$
$3\sqrt{a} + 2\sqrt[3]{a}$
Cannot be reduced.

EXAMPLES

$4\sqrt[3]{2} - 3\sqrt[3]{2} = \sqrt[3]{2}$

$5\sqrt{a} + 3\sqrt{a} = 8\sqrt{a}$

1. Add.

a) $\sqrt{5} + \sqrt{5}$

b) $6\sqrt[3]{2} - 4\sqrt[3]{2}$

c) $3\sqrt{7} - 7\sqrt{7}$

d) $3\sqrt{3} - 5\sqrt{3} + 4\sqrt{3}$

e) $5\sqrt[4]{7} + \sqrt[4]{7} - 3\sqrt[4]{7}$

f) $13\sqrt[3]{8} + 3\sqrt[3]{8} - 8\sqrt[3]{8}$

2. Calculate and reduce.

a) $\sqrt{a} + \sqrt{a} + \sqrt{a}$

b) $10\sqrt[5]{x} - 6\sqrt[5]{x}$

c) $3\sqrt[3]{a^2} + 3\sqrt[3]{a^2}$

d) $8\sqrt[5]{x} - 5\sqrt[5]{x} - \sqrt[5]{x}$

e) $6\sqrt{a^5} + 3\sqrt{a^5} - 8\sqrt{a^5}$

f) $7\sqrt[4]{x^3} - 5\sqrt[4]{x^3} + \sqrt[4]{x^3}$

Sample problems

Reduce.

$$\frac{\sqrt{5}}{2} - \frac{\sqrt{5}}{3} = \frac{3\sqrt{5}}{6} - \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{6}$$

$$\frac{4\sqrt[3]{a}}{5} - \frac{2\sqrt[3]{a}}{3} = \frac{12\sqrt[3]{a}}{15} - \frac{10\sqrt[3]{a}}{15} = \frac{2\sqrt[3]{a}}{15}$$

3. Convert the following fractions so that they share a common denominator, then simplify them.

a) $\dfrac{3\sqrt{3}}{4} - \dfrac{\sqrt{3}}{6}$

b) $\dfrac{\sqrt[3]{5}}{4} + \dfrac{\sqrt[3]{5}}{5}$

c) $\dfrac{\sqrt[4]{x}}{2} + \dfrac{\sqrt[4]{x}}{2}$

d) $\sqrt[5]{a} - \dfrac{2\sqrt[5]{a}}{3}$

e) $\dfrac{\sqrt{a}}{6} - \dfrac{\sqrt{a}}{9}$

f) $\dfrac{2\sqrt[5]{x}}{15} + \dfrac{\sqrt[5]{x}}{6}$

Sample problems

Reduce the powers.

$$\left(\frac{3\sqrt{5}}{2} - 4\right) - \left(\frac{\sqrt{5}}{4} - 1\right) = \frac{3\sqrt{5}}{2} - 4 - \frac{\sqrt{5}}{4} + 1 = \frac{6\sqrt{5}}{4} - \frac{\sqrt{5}}{4} - 4 + 1 = \frac{5\sqrt{5}}{4} - 3$$

$$(2\sqrt{a} - \sqrt{b}) + 3(\sqrt{a} - \sqrt{b}) = 2\sqrt{a} - \sqrt{b} + 3\sqrt{a} - 3\sqrt{b} = 5\sqrt{a} - 4\sqrt{b}$$

4. Remove the brackets and reduce the expressions.

a) $(5\sqrt[3]{2} + 2\sqrt[3]{3}) - (\sqrt[3]{2} + 3\sqrt[3]{3})$

b) $\left(\frac{2\sqrt{7}}{3} + 6\right) - \left(5 - \frac{\sqrt{7}}{3}\right)$

c) $(6\sqrt{x} - 3\sqrt{y}) - 2(2\sqrt{x} - \sqrt{y})$

d) $\frac{1}{2}(6\sqrt{a} - 4\sqrt{b}) - \frac{1}{3}(9\sqrt{a} - 6\sqrt{b})$

Products of radicals

- To multiply two radicals with the same index, you multiply the radicands.

$$\sqrt{5} \cdot \sqrt{4} = \sqrt{5 \cdot 4} = \sqrt{20}$$

- Radicals with different index numbers cannot be multiplied directly.

EXAMPLES

$\sqrt[5]{a^2} \cdot \sqrt[5]{a} = \sqrt[5]{a^2 \cdot a} = \sqrt[5]{a^3}$

$\sqrt[3]{x} \cdot \sqrt[3]{x^2} = \sqrt[3]{x \cdot x^2} = \sqrt[3]{x^3} = x$

5. Multiply and reduce if possible.

a) $\sqrt[3]{4} \cdot \sqrt[3]{2}$

b) $\sqrt[5]{5} \cdot \sqrt[5]{3}$

c) $\sqrt{12} \cdot \sqrt{3}$

d) $\sqrt[4]{9} \cdot \sqrt[4]{9}$

e) $\sqrt[3]{\frac{2}{9}} \cdot \sqrt[3]{\frac{4}{3}}$

f) $\sqrt[4]{8} \cdot \sqrt[4]{3}$

6. Multiply and reduce if possible.

a) $\sqrt[3]{x} \cdot \sqrt[3]{x}$

b) $\sqrt{a} \cdot \sqrt{a}$

c) $\sqrt[5]{x} \cdot \sqrt[5]{x^3}$

d) $\sqrt[4]{a^2} \cdot \sqrt[4]{a^2}$

e) $\sqrt[5]{x^3} \cdot \sqrt[5]{x}$

f) $\sqrt[5]{a^2b} \cdot \sqrt[5]{ab^2}$

BEAR IN MIND

$\sqrt[n]{a^n} = a$

Powers and radicals

The power of a radical can be simplified with the root's index if both are multiples of the same number.

$$(\sqrt[6]{5})^4 = (\sqrt[2\cdot 3]{5})^{2\cdot 2} = (\sqrt[3]{5})^2 = \sqrt[3]{5^2}$$

EXAMPLES

$\sqrt{4^6} = \sqrt[2]{4^{2\cdot 3}} = 4^3$

$\sqrt[6]{x^{10}} = \sqrt[2\cdot 3]{x^{2\cdot 5}} = \sqrt[3]{x^5}$

7. Simplify.

a) $\sqrt[4]{3^2}$

b) $\sqrt{5^{10}}$

c) $(\sqrt[6]{7})^8$

d) $(\sqrt[3]{x})^6$

e) $(\sqrt[4]{a})^4$

f) $\sqrt{x^6}$

5 RATIONAL AND IRRATIONAL NUMBERS

Sets of numbers

- Rational numbers ($\mathbb{Q}$) are those which can be expressed as fractions:

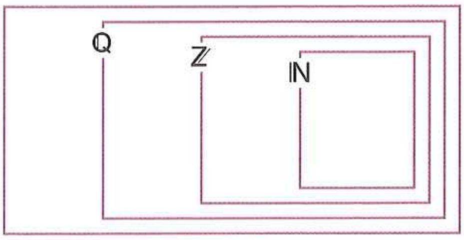
RATIONAL NUMBERS $\mathbb{Q}$
- WHOLE NUMBERS $\mathbb{Z}$
  - NATURAL NUMBERS ($\mathbb{N}$) $\rightarrow$ 0, 1, 2, 3, 4, 5, ...
  - NEGATIVE WHOLE NUMBERS $\rightarrow$ −1, −2, −3, −4, −5...
- FRACTIONS
  - EXACT DECIMALS $\rightarrow$ 0.2; 0.84; 1.27; ...
  - RECURRING DECIMALS $\rightarrow$ $0.\widehat{3}$; $1.\widehat{52}$; ...

- Irrational numbers are those which have infinite, non-recurring decimal numbers.

1. Circle the rational numbers and a draw a line through the whole numbers.

$\sqrt{2}$	−1.7	$1.\widehat{7}$	$-\sqrt{9}$	−13
$0.\widehat{28}$	$\sqrt{5}$	$\frac{10}{5}$	0.01	$-\frac{2}{3}$

2. Place the numbers to the right inside the appropriate boxes.



−5	$\frac{12}{3}$	0.1	$\sqrt{7}$
$\frac{\sqrt{2}}{2}$	$\frac{1}{3}$	$-\frac{10}{2}$	208

3. Write "T" if the statement is true and "F" if it is false.

a) Within the set of natural numbers there are an infinite number of natural digits. ⟶ ☐

b) The set of whole numbers encompasses that of natural numbers. ⟶ ☐

c) Every rational number is a whole number. ⟶ ☐

d) Every decimal number is a rational number. ⟶ ☐

e) The set of whole numbers is contained within the set of rational numbers. ⟶ ☐

4. Express the following numbers as fractions:

a) 0.001   b) 0.28   c) $0.\widehat{3}$

d) 5   e) $-0.\widehat{5}$ f) 200